MATH 220A

Midterm Exam, November 8, 2019

Instructions: 1 hour. You may use without proof results proved in Conway up to and including Chapter III. When using a result from the text, be sure to explicitly verify all hypotheses in it. Present your solutions clearly, with appropriate detail.

1. (15+15=30p) Compute the radii of convergence of the following power series:

$$(\mathbf{a})\quad \sum_{n=0}^{\infty} a^n z^{n^2} \qquad\qquad (\mathbf{b})\quad \sum_{n=1}^{\infty} n^2 a^n z^{n^2-1},$$

where $a \in \mathbb{C}$ and $a \neq 0$.

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2. (10+10+10=30p) Let $S(z)$ be a Möbius transformations such that S maps lines in $\mathbb{C}$ to lines. Determine all such $S(z)$ that also have k fixed points in $\mathbb{C}$, where

(**a**) $k = 2$ (**b**) $k = 1$ (**b**) $k = 0$.

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3. (30p) Let (X, d), (Ω, ρ) be metric spaces, and $G \subset X$, $\Delta \subset \Omega$ open subsets. A map $f\colon G \to \Delta$ is called *proper* if $f^{-1}(K) \subset G$ is compact for every compact $K \subset \Delta$. (The empty set is compact.) Suppose that $f\colon \overline{G} \to \overline{\Delta}$ is continuous and and its restriction to G is a proper map $G \to \Delta$. Show that $f(\partial G) \subset \partial\Delta$.
